Optimal Moving Grids for Time-Dependent Partial Differential Equations. A. J. Wathen. *RIACS, NASA Ames Research Center, Ames, Iowa 50010, USA.*

Various adaptive moving grid techniques for the numerical solution of time-dependent partial differential equations have been proposed. The precise criterion for grid motion varies, but most techniques will attempt to give grids on which the solution of the partial differential equation can be well represented. We investigate moving grids on which the solutions of the linear heat conduction and viscous Burgers' equation in one space dimension are optimally approximated. Precisely, we report the results of numerical calculations of optimal moving grids for piecewise linear finite element approximation of PDE solutions in the least-squares norm.

A Discrete Multiple Scales Analysis of a Discrete Version of the Korteweg–de Vries Equation. S. W. Schoombie. *University of the Orange Free State, Bloemfontein, South Africa.*

A more elaborate discrete multiple scales analysis than that used by Newell in 1977 is performed on the Zabusky–Kruskal discretization of the Korteweg–de Vries (KdV) equation. This eventually leads to a set of partial difference equations describing the modulational behavior of a small harmonic wave modulated by a slowly varying envelope. In the case of certain modes of the carrier wave, the multiple scales analysis breaks down, indicating that in these cases the numerical solution deviates in behavior from that of the KdV equation. Numerical experiments are reported which confirm this.

A Numerical Energy Conserving Method for the DNLS Equation. Tor Flå. *Institute of Mathematical and Physical Sciences, University of Tromsø, P.O. Box 953, N-9001 Tromsø, Norway.*

An implicit, numerical energy conserving method is developed for the derivative nonlinear Schrödinger (DNLS) equation for periodic boundary conditions. We find no numerical high frequency modulational instabilities in addition to the modulational instability from a linear analysis around a nonlinear state for the DNLS equation if the modulation is small and $(k_0 - a^2/2)^2 \tau < \pi$ (k_0 is the wavenumber and a the amplitude). The numerical scheme is used to follow the nonlinear behavior of the DNLS modulational instability. The numerical code is also tested by the evolution for one soliton initial data. These tests show that if the modulation is not small compared to the background wave amplitude, new nonlinear numerical instabilities are introduced.

A Relaxation Algorithm for Classical Paths as a Function of End Points: Application to the Semiclassical Propagator for Far-from-Caustic and Near-Caustic Conditions. A. G. Basile. *Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501, USA*; C. G. Gray. *Guelph–Waterloo Program for Graduate Work in Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1.*

We present a relaxation algorithm for obtaining the classical and non-classical paths from the boundary value problem with fixed initial and final positions and times on the path, and we discuss a technique for obtaining all paths connecting a given set of end points. From these paths, the action and other essential quantities entering the far-from-caustic and near-caustic expressions for the semiclassical propagator can be obtained. We illustrate with three one-dimensional examples—a time-dependent harmonic oscillator, a double-well anharmonic oscillator, and the repulsive $1/x^2$ potential—and find good agreement between the numerically calculated and exact paths where analytical results are available for comparison. We also find surprisingly good agreement between the semi-classical propagator and the exact propagator in case where the latter is available for comparison.

An Adaptive Mesh Refinement Method for Nonlinear Dispersive Wave Equations. Eric S. Fraga. *University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*; John Ll. Morris. *University of Dundee, Dundee DD1 4HN, Scotland.*

Adaptive mesh refinement techniques are often essential for solving nonlinear partial differential equations numerically. A new method for spatial grid refinement is developed and implemented. Several numerical experiments are performed to compare the method with results obtained using a uniform grid. The new method has the following properties: it is simple to implement and requires little modification of existing code to use; the solutions achieved as a result of using these methods prove to be accurate; and, the stability of the numerical methods is affected minimally. The effect of the grid refinement on essential properties of some of the equations, such as conservation, is minimized through the use of piecewise uniformity.

On Multigrid Solution of High-Reynolds Incompressible Entering Flows. A. Brandt. *The Weizmann Institute of Science, Rehovot 76100, Israel*; I. Yavneh. *Center for Nonlinear Studies and T-7, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA, and The Weizmann Institute of Science, Rehovot 76100, Israel.*

An approach is presented for effectively separating the solution process of the elliptic component of high-Reynolds incompressible steady entering flow, for which classical multigrid techniques are well-suited, from that of the non-elliptic part, for which other methods are more effective. It is shown by analysis and numerical calculations that such an approach is very effective in terms of asymptotic convergence as well as reduction of errors well below discretization level in a 1FMG algorithm.

Boundary Conditions for Direct Simulations of Compressible Viscous Flows. T. J. Poinsot. *Center for Turbulence Research, Stanford University, Stanford, California 94305, USA*; S. K. Lele. *NASA Ames Research Center, Moffett Field, California 94305, USA.*

Procedures to define boundary conditions for Navier–Stokes equations are discussed. A new formulation using characteristic wave relations through boundaries is derived for the Euler equations and generalized to the Navier–Stokes equations. The emphasis is on deriving boundary conditions compatible with modern non-dissipative algorithms used for direct simulations of turbulent flows. These methods have very low dispersion errors and require precise boundary conditions to avoid numerical instabilities and to control spurious wave reflections at the computational boundaries. The present formulation is an attempt to provide such conditions. Reflecting and non-reflecting boundary condition treatments are presented. Examples of practical implementations for inlet and outlet boundaries as well as slip- and no-slip walls are presented. The method applies to subsonic and supersonic flows. It is compared with a reference method based on extrapolation and partial use of Riemann invariants. Test cases described include a ducted shear layer, vortices propagating through boundaries, and Poiseuille flow. Although no mathematical proof of well-posedness is given, the method uses the correct number of boundary conditions required for well-posedness of the Navier–Stokes equations and the examples reveal that it provides a significant improvement over the reference method.